Warmups ① $[\ln(3x^2)]'$ (do it two ways)

② $(\arcsin(3p^2))'$

③ $\int \left(\frac{\sin(x)}{x} + \cos(x)\ln(x)\right) dx$

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① $[\ln(3x^2)]' = \frac{1}{3x^2}\cdot(6x) = \boxed{\frac{2}{x}}$. (method 1)

$[\ln(3) + 2\ln(x)]' = \boxed{\frac{2}{x}}$ (method 2)

$\ln(AB) = \ln(A) + \ln(B)$

$\ln(A^C) = C\cdot\ln(A)$

② $[\arcsin(3p^2)]'$

$= \frac{1}{\sqrt{1-(3p^2)^2}}\cdot(6p) = \boxed{\frac{6p}{\sqrt{1-9p^4}}}$

③ $\int \left(\frac{\sin(x)}{x} + \cos(x)\ln(x)\right) dx = \boxed{\sin(x)\ln(x) + C}$

Example

Compute $\frac{d}{dt}\left(\int_{t^2}^{4} \cos(e^{x^5})\,dx\right)$.

Recall 2FTC: $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$

$$\frac{d}{dt}\left(\int_{t^2}^{4}\cos(e^{x^5})\,dx\right) = -\frac{d}{dt}\left(\int_{4}^{t^2}\cos(e^{x^5})\,dx\right)$$

$$= -\cos\left(e^{(t^2)^5}\right)\cdot \underbrace{(t^2)'}_{2t}$$

$$= \boxed{-2t\cos\left(e^{t^{10}}\right)}$$

$$\begin{pmatrix}\text{2FTC}\\ \text{+ chain rule}\end{pmatrix} \quad \frac{d}{dx}\left(\int_a^{u(x)} f(t)\,dt\right) = f(u(x))\cdot u'(x)$$

<u>Example</u> Find $\frac{d}{dx}\left(\int_{2x}^{3x}\sin(t^2)\,dt\right)$

$$\frac{d}{dx}\left(\int_{2x}^{0}\sin(t^2)\,dt + \int_0^{3x}\sin(t^2)\,dt\right)$$

$$= -\frac{d}{dx}\left(\int_0^{2x}\sin(t^2)\,dt\right) + \frac{d}{dx}\left(\int_0^{3x}\sin(t^2)\,dt\right)$$

$$= -\sin\left((2x)^2\right)\cdot 2 + \sin\left((3x)^2\right)\cdot 3$$

$$= \boxed{-2\sin(4x^2) + 3\sin(9x^2).}$$

Another version of 2FTC:
If f is a continuous fcn, then if we define
$G(x) = \int_a^x f(t)\,dt$, then $G'(x) = f(x)$.

$\Longleftrightarrow \left(\int_a^x f(t)\,dt\right)' = f(x)$

$\Longleftrightarrow \frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x).$

Example

Find $\displaystyle\int_0^4 \sqrt{16-a^2}\,da$

$y=\sqrt{16-a^2}$

$y^2=16-a^2$

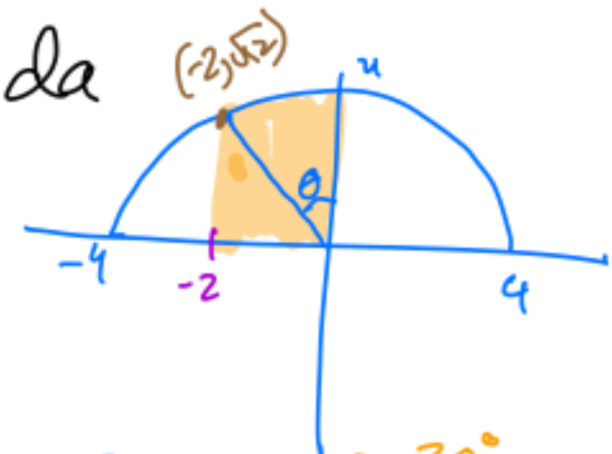
$a^2+y^2=16=R^2$

$= \frac{1}{4}(\pi r^2) = \frac{1}{4}(\pi \cdot 16) = \boxed{4\pi}.$

Find $\displaystyle\int_{-4}^{4} \sqrt{16-a^2}\,da =$

$\frac{1}{2}\pi R^2 = \frac{1}{2}16\pi = \boxed{8\pi}$

Find $\displaystyle\int_{-2}^{0} \sqrt{16-a^2}\,da$



$2\sqrt{3} = \sqrt{12}$

$\frac{\sqrt{3}}{2}$

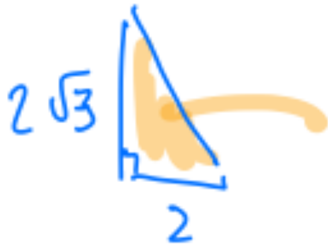
$\frac{1}{2}$

$a=-2$

$y=\sqrt{16-a^2}=\sqrt{16-4}=\sqrt{12}$

$\theta = 30^\circ.$

$$= \frac{30}{360}\cdot\left(\begin{matrix}\text{whole}\\ \text{disk}\\ \text{area}\end{matrix}\right) = \frac{1}{12}\pi R^2 = \frac{1}{12}\pi 4^2 = \frac{16\pi}{12} = \frac{4\pi}{3}.$$



$$= \frac{1}{2}bh = \frac{1}{2}2(2\sqrt{3}) = 2\sqrt{3}$$

Ans.

$$\boxed{\int_{-2}^{0}\sqrt{16-a^2}\,da = \frac{4\pi}{3}+2\sqrt{3}}$$

Could also do this by $a = 4\sin\theta$ substitution.

Geometric Series

$$a + ar + ar^2 + \dots + ar^n$$

$$= \sum_{k=0}^{n} a\cdot r^k = \frac{a(1-r^{n+1})}{1-r}.$$

$$100 + 100(1.1) + 100(1.1)^2 + \dots + 100(1.1)^{11}$$

$$= \sum_{k=0}^{11} 100\cdot(1.1)^k = \frac{a(1-r^{n+1})}{1-r} = \frac{100\,(1-1.1^{12})}{1-1.1}$$

$a = 100 = 1^{st}$ term, r = ratio, $(n+1) = \#$ of terms

$$= \frac{100\left(1-1.1^{12}\right)}{-.1} = \frac{100(1-3.1384)}{-0.1}$$

$$= \frac{100(-2.1384)}{-0.1} = \$2138.40$$